Learning Trajectories in Online Mathematics Courses

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RESEARCH INSTITUTE

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- Networks – Develop human and web-based applications and infrastructures for sharing information and implementing K-12 online and blended learning best practices.

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Executive Summary

The present research has devoted attention to a long-standing problem: how to better serve students who take K-12 online mathematics courses. Drawing on various learning analytics using learning management system (LMS) data, previous research has suggested that for mathematics learning, the variable of logged-in-duration predicted course outcomes positively, whereas the logged-in-frequency showed a counter-intuitive result: the more often students logged-in, the less likely they were to achieve. However, the previous research, which focused on such aggregate variables, could not adequately capture patterns of learning progress over time in online mathematics courses. Moreover, given that research on predictors that consistently account for success or failure in online mathematics learning is still in its early stages, a search for alternative approaches in terms of methodology should ensue.

In this context, the current study attempted to fill the important research gap identified above by using growth-mixture modeling (GMM). Growth modeling enabled the study to encompass both the types of variation in student behaviors across students as well as the behaviors of a single student over time. Additionally, mixture modeling fit the needs of the study as it was based not on a (typical) variable-oriented approach but rather a person-oriented approach, which is adequate in particular when there is a lack of certainty about dimensional identity, i.e., how a set of variables defines and confirms a trait of behavior or development.

The GMM of LMS data from 1,828 enrollments in Algebra I, Algebra II, AP Calculus, AP Statistics, Calculus, Consumer Math, Foundation Math, Geometry, Pre-Algebra, Pre-Calculus, Probability and Statistics, and Trigonometry revealed four meaningful sub-groups (termed latent classes) of learners in online mathematics learning over the time period of a semester: (1) nearly linear trajectory (73%); (2) exponential growth (14%); (3) hardly any growth (12%); and (4) early rapid growth (1%). Chi-square testing and its post-hoc analysis using standardized residuals highlighted some group differences in individual latent classes among semesters, locale codes, enrollment reasons, and course types. With those results, implications for practitioners and researchers were discussed from the perspective of the pacing guide, student autonomy, and evidence-based practices in secondary mathematics learning.

Introduction

Despite their large numbers of enrollments, online K-12 mathematics courses have among the worst student outcomes. In the state of Michigan, this phenomenon has been carefully documented in annual reports on K-12 virtual learning effectiveness (Freidhoff, 2015-2018; Freidhoff, DeBruler, Kennedy, 2014). These show that while the total number of virtual enrollments in the state grew continually between the 2010-11 and 2016-17 academic years, mathematics courses – which consistently accounted for around one-fifth of all such enrollments – had the lowest pass rates among high-enrollment courses from 2011-12 to 2014-15 and the lowest among all course types in 2015-16 and 2016-17.

Despite these and other disappointing results, few researchers have offered systematic explanations of students' engagement or learning behaviors in online mathematics courses. Among them, Liu and Cavanaugh (2011) found that the more often students attempted to log into an LMS, the lower their course grades in Algebra 2 and Geometry 1 were. For logged-in duration, however, the same study found the reverse effect: the longer students stayed in the LMS, the better they performed in both subjects, as well as in several other mathematics courses. Unfortunately, Liu and Cavanaugh did not provide descriptive distributions of these two behavioral indicators. Nevertheless, the counterintuitive association between students' levels of course-related online activity and their outcomes was reiterated by Hung, Hsu, and Rice (2012) who found that mathematics courses were associated with both a relatively high average frequency of total clicks per student and with poor outcomes in comparison to other subject areas.

In this line of research, students' grade histories are worthy of attention as another behavioral indicator to be explored. This is especially so in the case of adaptive-release courses, i.e., those in which learners are required to complete tasks to a certain standard (typically, a grade of 60% or more) before they are allowed to begin the next lesson. In their exploration of an adaptive-release Algebra 1 course, Lowes and Lin (2017) created heat maps of the course points the students earned each week, along with the number of weeks they fell behind, based on the pacing guide. Descriptive analyses of these heat maps highlighted how much variability there was across students. Overall, students who passed the course stayed closer to the pacing guide than students who failed by earning course points consistently throughout the duration of the course; but the authors noted that a two-level grouping variable (i.e., passing vs. failing) provided insufficient information about the unique learning pathways of smaller sub-groups of learners. Accordingly, they employed cluster analysis with more elaborate grouping variables, namely (1) good pacing and good grades, (2) poor pacing and good grades, (3) good pacing and poor grades, and (4) poor pacing and poor grades. Again, however, the findings provided a clear picture only of the best and worst clusters, i.e., (1) and (4) and again implied variability in the middle, i.e., (2) and (3).

Accordingly, the present study is an attempt to fill the important research gap identified above, using data from all middle- and high school-level mathematics courses offered by *Michigan Virtual School* in the fall and spring semesters of the 2015-16 academic year. The final sample included 1,828 enrollments in Algebra I, Algebra II, AP Calculus, AP Statistics, Calculus, Consumer Math, Foundation Math, Geometry, Pre-Algebra, Pre-Calculus, Probability and Statistics, and Trigonometry.

The present research proceeds from Lowes and Lin's (2017) conclusion that aggregate variables such as total login frequency cannot adequately capture patterns of learning progress in online mathematics courses. Instead this study dichotomizes variation in student behaviors into differences across students and differences over time by using GMM as its analytical approach with month-by-month course scores. The existing literature's failure to identify either learning profiles or meaningful subgroups of K-12 online learners justifies the use of GMM, in which neither the number of profiles nor their descriptions are determined *a priori*, but rather are derived from clusters in the data. The application of GMM is also appropriate, given that research on predictors that consistently account for success or failure in online mathematics learning is still in its early stages. That is, GMM is a person-oriented approach rather than a variable-oriented one, and the former is often selected when there is uncertainty about dimensional identity, i.e., how a set of variables defines and confirms a trait of behavior or development (Bergman & Trost, 2006).

The current research was guided by the following three research questions:

1. What is the best-fitting model of students' learning trajectories in online mathematics courses, as derived from their month-by-month course scores?
2. How do data-driven subgroups of online mathematics learners differ along with the categorical variables of semester, gender, enrollment reason, locale code, and course type?
3. How do the data-driven subgroups of online mathematics learners differ in terms of their final course outcomes?

Methods

Growth Mixture Modeling

Growth models examine how individual's behaviors change over time and thus require data collected via repeated measures. However, repetition by itself is not a guarantee that growth modeling is an appropriate technique; its design must also consider whether occasion-to-occasion changes are large enough. For example, month-by-month changes in human height would not be a suitable case, given the data's stability (Grimm, Ram, & Estabrook, 2017). The present study used LMS data comprising records of students' learning outcomes (i.e., monthly course grades) and activities obtained on multiple occasions throughout each semester and readily transformed into a format suitable to the sensitivity of the measurement.

The target online learning environment allowed for considerable autonomy. Proceeding at their own pace, students logged into their courses to complete assignments, collaboration and discussion tasks, quizzes, and tests, then were awarded points for each of these activities. Being dependent on their learning progression, students' cumulative scores differed widely, both across individuals and across time-points.

Notably, conventional growth modeling assumes that its sample is drawn from a single population, that the baseline between-person differences and the growth rate over time exist along a continuum, and that such differences and growth rate are both normally distributed (Jung & Wickrama, 2008). However, an examination of behaviors and performance in online courses cannot rely on the assumption of a single growth trajectory for various subsets of students. For instance, there is evidence for gender differences in both course engagement and performance (Lowes, Lin, & Kinghorn, 2016). The performance of students who take a given online course to recover credits they failed to earn previously is also relatively poor, in comparison to that of students taking the same course for other reasons, e.g., that it was not offered by their brick-and-mortar schools (Kwon, 2017).

Unlike its conventional counterpart, GMM takes into account unobserved group differences in changes of individuals' attributes over time and thus enables researchers to identify and explore the qualitatively distinct trajectories of *latent classes* (i.e., subgroups), the number and identity of which are specified by the modeling process. However, variation in model estimation is possible if one specifies the number of latent classes and/or defines the trajectory type, e.g., as linear, quadratic, or cubic growth. The final procedure is to select the best-fitting model in light of model-selection criteria. For the current study, all GMM models were created in Mplus version 8 (Muthén & Muthén, 1998-2017).

Follow-up Analyses

The characteristics of individual latent classes from the best-fitting model were explored through chi-square testing. Two-way contingency tables were constructed with the latent class from that model and the other categorical variables, including Semester (fall or spring), Gender, Locale, (city, suburb, town, or rural), AP course (AP or Non-AP), Algebra course (Algebra or Non-Algebra), Enrollment Reason (unavailable course, schedule conflict, learning preference, or credit recovery), and Course Completion Status (passed, failed, or withdrawn).

When a chi-square test indicated a statistically significant difference in the relative frequency of a latent-class variable over individual group variables, a post-hoc test – whether or not a particular cell's proportion differed from others in the two-way contingency table – was performed by calculating and observing adjusted residuals for each cell. Post-hoc tests of this kind have the potential to increase the rate of type I errors (false discoveries) because they involve simultaneous testing of multiple hypotheses: in this case, 16 tests, from the table of four classes × four locales. To address this familywise error rate, an absolute value greater than three was applied to determining statistical significance, which is more parsimonious than using a confidence level of 99% (i.e., a p value < 0.001). All calculations were conducted in Stata version 14 (Stata Corp., 2015).

Findings

This section sets forth the present study's findings, beginning with its data-analysis process and proceeding via model estimation and selection, interpretation of the best-fitting model, and follow-up analysis of that model's results to observation of course outcomes.

Model Selection

An exploratory approach was adopted in which multiple scenarios were estimated depending on the number of classes and the shapes of trajectories until satisfactory solutions were determined. Five occasions, i.e., months, of measurements were sufficient to allow examination of linear, quadratic, and cubic time factors; and Table 1 presents the associated model-fit indices.

Table 1. Fit of Growth Mixture Models

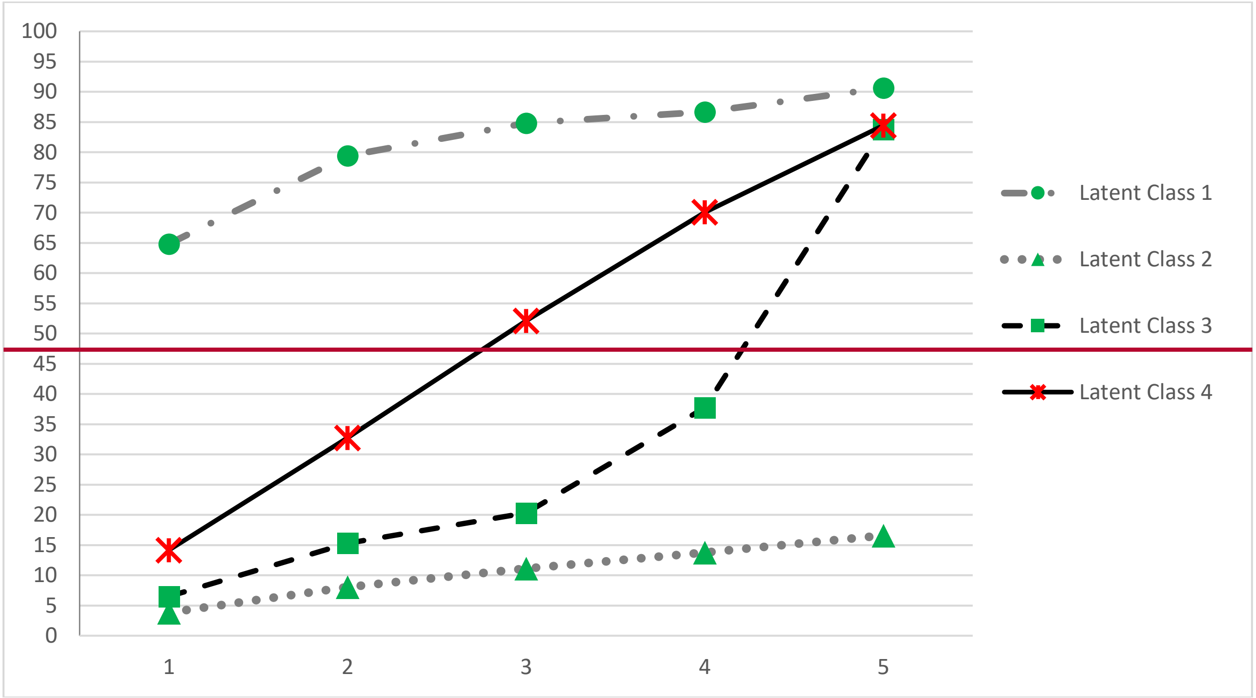
Model	Latent Class	Likelihood	Sample Adjusted BIC[1]	Entropy	Lo-Mendell-Rubin Test		Proportion of Individuals in Class				
					2LL	p value	1	2	3	4	5
Linear GMM											
Model 1.1	1	-36732.808	73517.625				1.00				
Model 1.2	2	-36446.301	72961.946	0.995	554.557	0.0076	0.98	0.02			
Model 1.3	3	-36274.439	72635.559	0.915	332.652	0.2275	0.01	0.86	0.13		
Quadratic GMM											
Model 2.1	1	-35863.511	71800.701				1.00				
Model 2.2	2	-35067.339	70230.027	0.984	1551.043	0.0000	0.12	0.88			
Model 2.3	3	-34781.780	69680.578	0.987	556.306	0.0305	0.02	0.86	0.12		
Model 2.4	4	-34591.291	69321.271	0.922	371.096	0.0046	0.15	0.01	0.12	0.71	
Model 2.5	5	-34537.772	69235.902	0.867	104.262	0.6452	0.15	0.01	0.59	0.12	0.13
Cubic GMM											
Model 3.1	1	-35809.297	71718.276				1.00				
Model 3.2	2	-35006.012	70133.377	0.985	1564.899	0.0000	0.12	0.88			
Model 3.3	3	-34721.683	69586.388	0.988	553.910	0.0027	0.12	0.86	0.01		
Model 3.4	4	-34557.075	69278.842	0.913	320.677	0.0000	0.01	0.12	0.14	0.73	
Model 3.5	5	-34431.121	69048.605	0.881	245.373	0.2695	0.01	0.14	0.60	0.12	0.12

Note[1]: Bayesian Information Criterion.

Lo-Mendell-Rubin testing quantifies the likelihood that a dataset can be described by a model with one less class; a *p* value smaller than 0.05 indicates that the model with one more class has a significantly better fit to the data than its less-complex counterpart (Lo, Mendell, & Rubin, 2001). Therefore, within each of the three above-mentioned general types of trajectory, model estimation was stopped once a Lo-Mendell-Rubin test's *p* value was less than 0.05. For instance, Model 1.3 has a *p* value of 0.23, which suggests that a model of a linear trajectory with three latent classes has no significant model-fit advantage over a model of the same trajectory with two latent classes. Accordingly, the researchers adopted the two-class model of this trajectory, no further modeling of which was required.

The Bayesian Information Criterion (BIC) (Schwarz, 1978) was used to select the best of the present study's 13 models. Specifically, a smaller BIC indicates a better model, and a BIC difference of 10 or more serves as evidence that one model is better than another (Raftery, 1995). The two lowest overall BIC values were achieved by models that both embedded four latent classes, one (Model 2.4) with a quadratic trajectory, and the other (Model 3.4) with a cubic one. A BIC difference of 42 between these two models clearly indicated that the cubic-trajectory model was the better of the two.

Lastly, entropy value quantifies the uncertainty that attends the classification of subjects into latent classes, with 0 corresponding to randomness and 1 to a perfect classification (Celeux & Soromenho, 1996). All things considered, then, the researcher's conclusion regarding model selection can be stated thus: the best fit to the data was provided by a cubic-trajectory growth model that contained four subgroups and had an entropy value of 0.91. Figure 1 presents model-estimated average course scores across five time-points for the four latent classes' members. The full model results are shown in Appendix A.



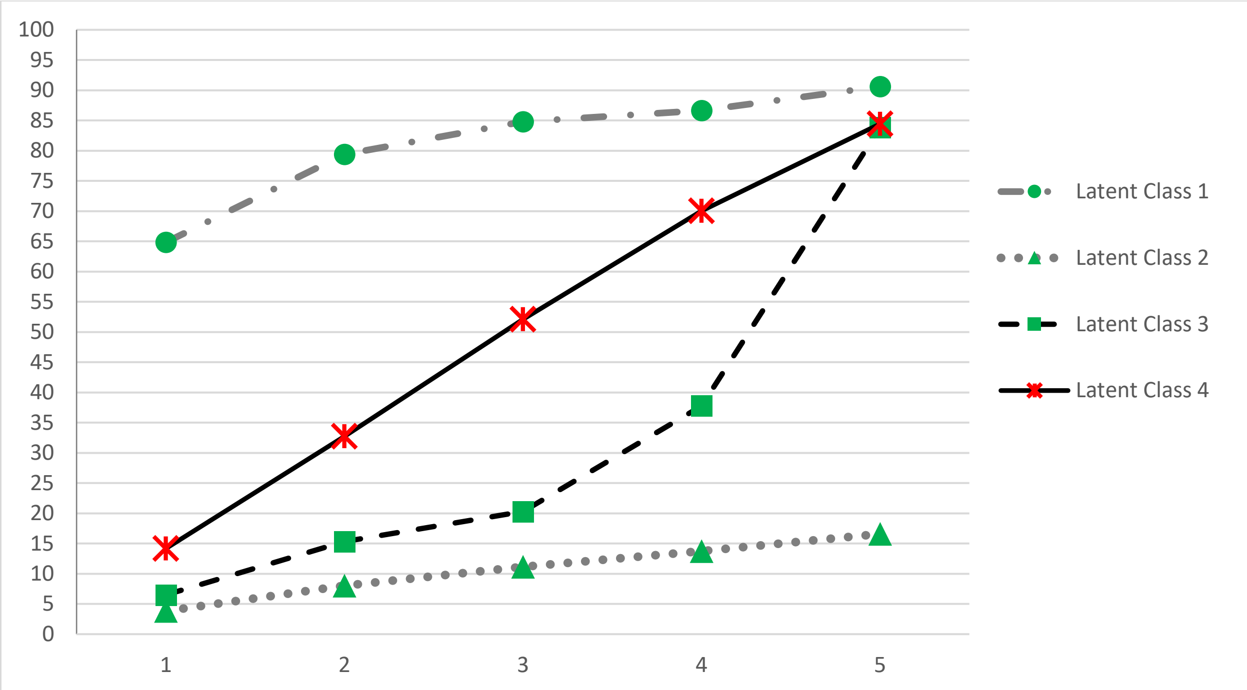


Figure 1. Trajectories of the final four-class cubic model.

The largest class (i.e., Class 4; refer to the solid line in Figure 1) made up approximately three-quarters of enrollments in the dataset. Its trajectory was nearly linear: a gradual increase from the first month to the last. At 14% of the sample, the next largest class (Class 3, denoted by the dashed line) was characterized by exponential growth, i.e., a much steeper increase in the members' scores as the end of each semester approached. For another 12% of the enrollments (Class 2, denoted by the dotted line) there was hardly any growth over either 20-week semester. Finally, just 1.4% of the enrollments (Class 1; refer to the dash dotted line) showed strong early achievement, earning 60% or more of the total course points during just the first two months of each five-month semester.

Follow-up analyses examined individual classes' characteristics in detail. The results are presented in Table 2, which reports three types of statistics: (1) the proportion made up by members of each class within a subset (e.g., Class 1 members of the subset *fall semester*); (2) chi-square statistics for individual contingency tables (e.g., semester × class); and (3) post-hoc test results using standardized residual estimates, in those cases where chi-square testing indicated significant group difference(s) within a given contingency table.

In the same table, the standardized residual estimates are reported in italics. Positive values indicate a greater likelihood that a person is a member of the pertinent class. Cells with no statistical significance are denoted *< 3*. For example, the semester × class results shown suggest that students in the fall semester were less likely to be members of Class 1 than those students in the spring semester. With regard to students' likelihood of being in Class 2 or Class 4, however, there was no difference between the two semesters. Taken together, this implies that the inter-semester differences associated with all latent classes in combination (i.e., significant chi-square statistics) are in fact attributable to group differences in Classes 1 and 3 only.

Table 2. Characteristics of Members Following the Four Latent Trajectories

Category	Total 1,828	Class 1 1.3%	Class 2 12.2%	Class 3 13.8%	Class 4 72.7%
Semester					
Fall	935	0.4%	13.1%	16.3%	70.3%
Spring	893	2.2%	11.3%	11.3%	75.1%
	X^2 (3) = 22.1***	*Fall -3.4 Spring +3.4*	*< 3*	*Fall +3.6 Spring -3.6*	*< 3*
Gender					
Female	938	1.3%	10.5%	15.0%	73.2%
Male	889	1.4%	14.1%	12.6%	72%
	X^2 (3) = 6.9	*n/a*	*n/a*	*n/a*	*n/a*
Locale					
City	142	0.7%	12.7%	19.0%	67.6%
Rural	679	1.0%	14.6%	11.8%	72.6%
Suburb	557	0.7%	13.1%	14.4%	71.8%
Town	312	3.2%	7.4%	9.6%	79.8%
	X^2 (9)=30.9***	*Town +3.3*	*Town -3.1*	*< 3*	*< 3*
Enrollment Reason					
Unavailable	789	2.5%	7.2%	11.4%	78.8%
Recovery	53	0%	35.9%	20.8%	43.4%
Preference	376	0.3%	18.1%	15.7%	66%
Scheduling	268	0.4%	8.6%	16.4%	74.6%
	X^2 (9)=90***	*Unavail. +3.6*	*Unavail. -5.2 Recovery +5.8 Prefer. +4.9*	*< 3*	*Unavail. +4.9 Recovery -5.1 Prefer. -3.9*
Course 1					
AP	340	0.3%	4.4%	0.6%	94.7%
Not-AP	1,488	1.6%	14.0%	16.9%	67.6%
	X^2 (3) =105***	*< 3*	*AP -4.9 Non-AP +4.9*	*AP -7.8 Not-AP +7.8*	*AP +10.1 Non-AP -10.1*
Course 2					
Algebra	370	0.3%	21.9%	17.8%	60%
Not-alg.	1,458	1.6%	9.7%	12.8%	75.9%
	X^2 (3) =55.2***	*< 3*	*Alg. +6.4 Non-Alg. -6.4*	*< 3*	*Alg. -6.1 Non-alg. +6.1*
Course 3					
Geometry	140	1.4%	24.3%	22.1%	52.1%
Not-geo.	1,688	1.3%	11.2%	13.2%	74.4%
	X^2 (3) =34.5***	*< 3*	*Geo. +4.5 Not-geo. -4.5*	*< 3*	*Geo. -5.7 Not-geo. +5.7*
Course 4					
Calculus	264	0.4%	8.7%	23.1%	67.8%
Not-calc.	1,828	1.5%	12.8%	12.3%	73.5%
	X^2 (3) =25.3***	*< 3*	*< 3*	*Calc. +4.7 Not-calc. -4.7*	*< 3*

As compared to the average prevalence of each latent class across the full data-collection period, the proportions of both Class 1 and Class 4 individuals were higher in the spring semester, whereas Class 3 students were more commonly observed in the fall. Statistical tests confirmed that more students were likely to exhibit early rapid growth (i.e., to be members of Class 1) in the spring, and to complete their coursework in a late rush (Class 3) in the fall. From the contingency table of gender × latent class, it can be seen that a relatively high proportion of Class 2 were males; however, this group difference was not statistically significant.

The two noteworthy results that emerged from the locale × latent class contingency table were relatively high proportions of (1) Class 3 membership among students from cities, and (2) Class 1 and Class 4 membership among those from towns. Conversely, Class 2 and Class 3 membership among town dwellers was lower than the average. However, in the case of students from city locales, only Class 4 membership was noticeably rare, with Class 1 and Class 2 about average. Statistical testing reaffirmed that in a town locale, a student was more likely to be in Class 1 and less likely to be in Class 2. Any difference related to city-dwelling students was not statistically significant.

When analyzing the variable *enrollment reason*, the LMS's open-ended choice (i.e., "Other") was excluded. Among the four remaining answer choices – "Unavailable at the local school", "Credit recovery", "Learning preference of the student", and "Scheduling conflict" – there was a group difference in class × enrollment reason. That is, post-hoc testing revealed that students whose online course-taking supplemented their local schools' course offerings were more likely than others to be members of Class 1 or Class 4 and less likely to be in Class 2; whereas students who were attempting to recover credits or who cited personal learning preferences were disproportionately likely to be Class 2 members and unlikely to be Class 4 members.

Interestingly, the AP course × latent class contingency table revealed that around 95% of students in AP courses were assigned to Class 4 by the GMM, which engendered statistical differences among Classes 2, 3, and 4 by the AP course group variable. In other words, AP students were not only more likely to exhibit a gradual, consistent trajectory over a given semester, but also less likely to exhibit an overall low-growth or exponential-growth one than those in other types of mathematics courses. Students in other advanced-level courses, such as Pre-calculus and Calculus, tended to be members of Class 3, i.e., to have an exponential trajectory throughout each semester. In stark contrast, students in graduation requirement courses, such as Algebra, Pre-algebra and Geometry, were more likely than others to be members of Class 2 and less likely to be in Class 4.

Because the final course outcomes of students who exhibited either low growth throughout the semester or early rapid growth could simply be intuited (see Appendix B), this study's outcome analysis focused exclusively on Classes 3 and 4. Accordingly, a comparison of final outcomes across the students with nearly linear trajectories and those with exponential-growth trajectories is presented in Table 3, which shows that the proportions of fails and passes in Classes 3 and 4 are quite consistent with those classes' respective percentages of all cases examined. For instance, Class 4, which included 84% of 1,580 enrollment records, accounted for 85.7% of course failures and 84% of successful course completions. Moreover, chi-square tests found no differences in final outcomes between the linear-growth and exponential-growth groups.

Table 3. Course Outcomes Comparison between Class 3 and Class 4

Completion Status	Total 1,580	Class 3 16%	Class 4 84%
Withdrawn	1	100%	0%
Failed	35	14.3%	85.7%
Passed	1,544	16%	84%
	X^2 (2) = 5.3	*n/a*	*n/a*

Discussion

This study's investigation of meaningful subgroups of learners in online mathematics courses, based on their semester-long learning trajectories, has considerable potential to inform the design and implementation of more effective online mathematics courses. The largest of the four data-driven latent classes was distinguished by a nearly linear growth trajectory; and pacing guides are generally designed to guide students to complete assignments consistently and persistently following a linear growth projection. Thus, the linear-growth subgroup/trajectory can be construed as those students who attempted to, and also were capable of, staying close to the pacing guides that were provided to them. Moreover, such students had a pass rate of 98% and no mid-semester dropouts.

Notably, AP courses, and courses not offered by students' local schools, were more likely than others to include this successful student type. Additionally, over half of AP course enrollments indicated the enrollment reason "Unavailable courses"; when the "Other" category cases were removed, this rose to 70%. Taken together, these findings point to an association between steady pacing and AP status, which could reflect MVS's instructional practices – specifically, more regular and strict prompting of its AP students to adhere to their pacing guides. Therefore, explicit and implicit instructional practices that are closely aligned with established pacing guides could boost an individual's odds of sustaining a steady pace, and in turn, his/her success in the course.

Nevertheless, the present findings should not be taken to mean that key features of online learning – namely, "any time, any place, any path, and any pace" – should be in any way watered down. Rather, it should be borne firmly in mind that no differences were found between the final outcomes of the linear-growth and exponential-growth groups. The latter, despite its slow start during the first half of each semester, was capable of reaching its target points and had the same pass rate (98%) with only one dropout. In other words, students could succeed academically despite marked variation in their self-determined pathways and pacing, even in advanced courses such as calculus. Accordingly, students who take advanced-level mathematics courses may benefit strongly from learner autonomy, i.e., being encouraged to take responsibility for their own learning and managing their time efficiently, without massive or fundamental change to current instructional practices and support structures being necessary.

However, the same cannot be said of some other learner groups, including the 12% of the current study's sample who were characterized by a slow start and low growth. Those students were more likely to be enrolled in foundation courses, including Algebra and Geometry, and to indicate that their enrollment reasons were credit recovery or personal learning preferences. This unpromising trajectory may be attributable to two distinct types of failure: lack of timely engagement and insufficient content mastery.

With regard to the first, prior studies' findings and recommendations that tend to be supported by the present study's empirical evidence include that (1) timely engagement is closely connected with mastery goal orientation, but not with performance (Strunk, Cho, Steele, & Bridges, 2013); (2) seeking help not only from formal sources (e.g., instructors and mentors) but also from informal ones (e.g., peers and others) needs to be emphasized and overtly taught as an online-learning strategy (Goda et al., 2013); and (3) embedding volition support into course content and/or instructional practices enhances effort regulation and course performance (Kim & Bennekin, 2016). Therefore, the study findings call attention to a question: How or how well could such elements be embedded into course designs or instructional practices in order for online mathematics learning environments to fit the needs of those at-risk learner groups better.

Regarding content mastery, *MVS*'s current course content should be reviewed in light of the conclusions of Slavin, Lakem, and Groff's (2009) meta-analysis of effective programs in middle and high school mathematics: specifically, that the best performance is associated with (1) programs in which cooperative learning, metacognitive-strategy training, and learning for mastery are interwoven; and (2) cooperative learning programs in which heterogeneous teams help each other to master content via a cycle of teaching, teamwork, individual assessment, and recognition based on teamwork. This conclusion points to several promising applications for future research to see how applicable those elements recommended by the literature on secondary mathematics learning are to particular online mathematics learning contexts.

Lastly, a vital question for instructors and mentors may be how to distinguish would be latent Class 2 members from Class 3 members before the final month when it could be too late for struggling students to complete the course successfully. While both classes shared slow starts, the at-risk student group never exceeded 10 percent points during the first two months whereas, those in Class 3 tended to have earned about 15% of the course points. There was also evidence of separation appearing in the third month (the midpoint of the semester) where Class 3 had about 20% of the course points and Class 1 only half that percentage. These percentage markers may help those supporting online learners better gauge which trajectory a student is most likely on.

It is also notable that while Class 3 members tend to be successful in earning credit for their online course, their overall achievement seems to reach only the mid-80s or, in most schools, a B grade. Given that this trajectory tended to be seen more often in advanced mathematics courses and presuming that upper-level mathematics courses would tend to have the stronger mathematics students in them, these students may be underestimating how much work they can successfully complete in the final month. For instance, the increase in scores from the end of the fourth month to the end of the fifth month was about 50 percentage points. This would mean that students wanting eventually to earn an A in their math course should have earned 40 percent or more of the courses points by the end of the fourth month. This interim benchmark may help students pace themselves a little better in the first four months and be more likely to achieve their end goal.

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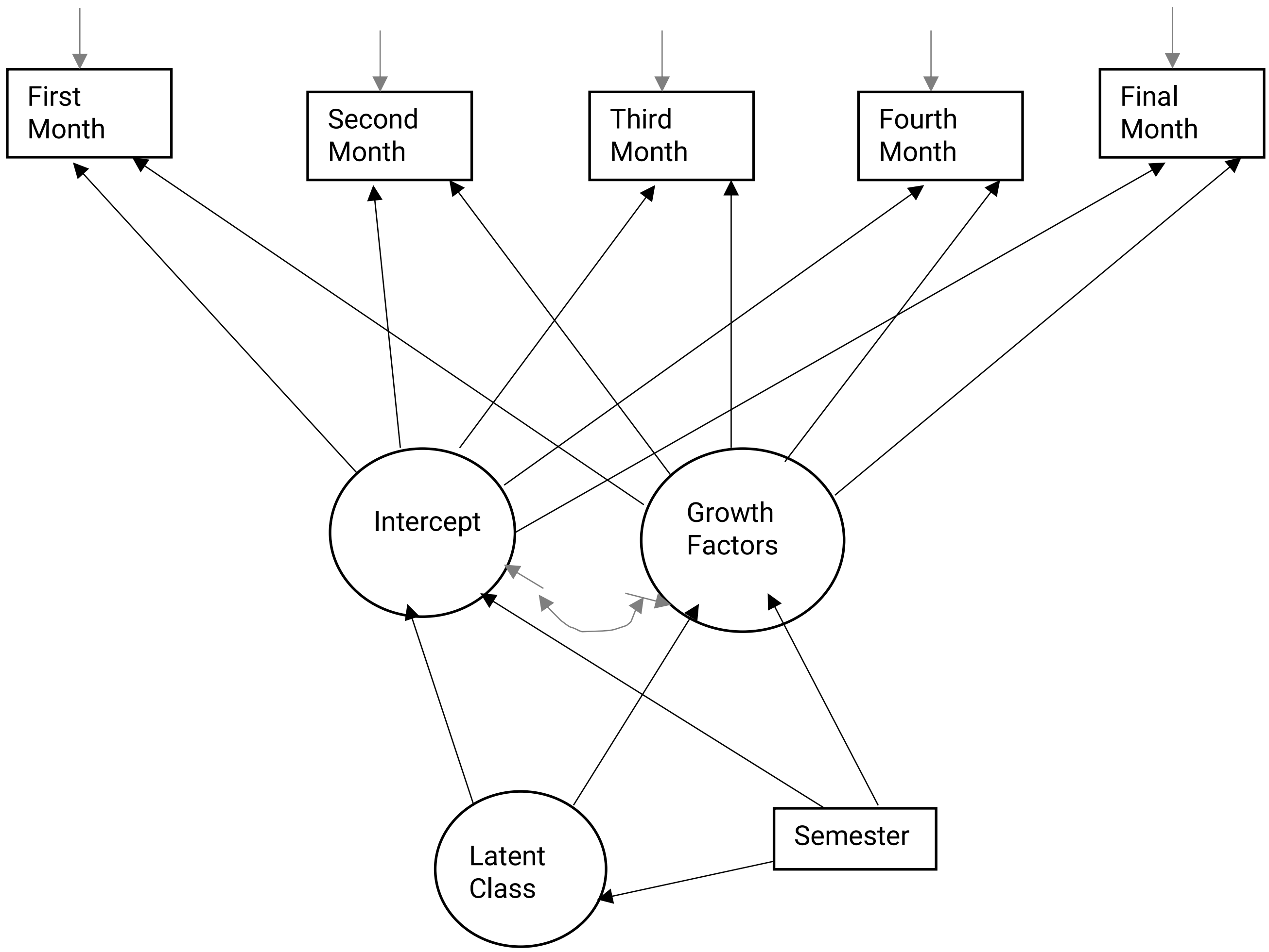
Appendix A. Best-Fit Model Results (Cubic Growth with 4-Class)

Diagram of Associations in the Model



	Estimate/Standard Error (*p* value)			
	Intercept	Slope	Quadratic	Cubic
Class 1 (1.3%)	49.45/5.3 (0)	25.83/5.24 (0)	-12.38/2.74 (0)	2.13/0.4 (0)
Class 2 (12.2%)	-8.28/0.84 (0)	8.78/1.61 (0)	-4.82/1.04 (0)	1.05/0.18 (0)
Class 3 (13.8%)	-5.38/0.9 (0)	19.83/1.81 (0)	-13.87/1.33 (0)	3.62/0.25 (0)
Class 4 (72.7%)	1.62/0.69 (0.019)	21.4/1.48 (0)	-2.6/0.96 (0.007)	0.61/0.17 (0)
Semester	8.36/0.55 (0)	-2.62/1.03 (0.011)	2.71/0.68 (0)	-0.65/0.12 (0)
Intercept	--	106.2/28.59 (0)	-48.25/12.93 (0)	5.44/1.71 (0.001)
Slope	--	--	-106.15/15.37 (0)	13.46/2.34 (0)
Quadratic	--	--	--	-9.51/1.65 (0)
Residual Variances				
First Month		36. 7/17.17 (0.033)		
Second Month		-2.22/7.82 (0.776)		
Third Month		68.9/7.42 (0)		
Fourth Month		64.94/8 (0)		
Final Month		9.87/18.51 (0.594)		
Intercept		32.23/16.47 (0.05)		
Slope		196.91/28.67 (0)		
Quadratic		67.16/10.43 (0)		
Cubic		1.44/0.26 (0)		

Appendix B. Summary of Final Course Outcomes (Cubic Growth with 4-Class)

	Class 1	Class 2	Class 3	Class 4
Passed	100%	0.9%	97.6%	97.7%
Failed	0%	87%	2%	2.3%
Withdraw	0%	12.1%	0.4%	0%
Total	24	223	253	1,327

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